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A STUDY ON NON-LINEAR SINGULAR SYSTEMS FROM FLUID DYNAMICS USING LEAPFROG METHOD

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ABSTRACT

This paper deals with the non- Linear singular system from fluid dynamics. The results (approximate solutions) obtained using single-term Haar wavelet series and Leapfrog methods are compared with the exact solutions of the non-Linear singular system from fluid dynamics. It is found that the solution obtained using Leapfrog method is closer to the original solution of the non-Linear singular system from fluid dynamics. The high accuracy and the wide applicability of Leapfrog method approach will be demonstrated with numerical examples.

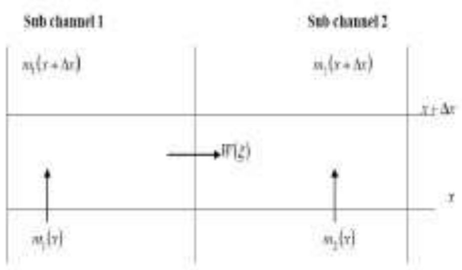
KEYWORDS: non- Linear singular system, Haar wavelet series, Leapfrog methods and fluid dynamics.

INTRODUCTION

A general numerical procedure for their solution has not previously existed. Hence it is important to understand the structure of such systems and develop efficient methods for solving them. Conventional methods, such as Euler, Taylor series and Adams-Moultan methods, are restricted to a very small step size in order to obtain a stable solution, which naturally require much computer time. Many new methods have been developed to overcome this step-size constraint imposed by numerical stability, and these are reviewed by Butcher [23, 24] and Murugesan et al. [97, 101]. Recently, Balachandran and Murugesan [14] obtained the numerical solution of a singular non-linear system from fluid dynamics. Murugesan et al. [101] analysis of non-Linear singular system from fluid dynamics using extended Runge-Kutta methods. S. Sekar et al. [149] analysis the same non-Linear singular system from uid dynamics using single-term Haar wavelet series method. The point to be noted is that the singular non-linear systems are much more did cult to solve than the linear singular systems. Therefore, many authors have tried various transform methods to overcome these difficulties. In this chapter, Leapfrog method is introduced to solve these non-Linear singular systems from fluid dynamics with more accuracy.

REPRESENTATION OF EQUATIONS OF OWN AS A NON- LINEAR SYSTEM

The simplified model consists of two connected sub channels filled with a steadily flowing fluid. Control volumes and flow variables for the system are shown in Figure 8.1. Here, mi, represents the axial mass ow rate in sub channel i and w represents the cross- flow rate per unit length, assumed positive if the flow is from sub channel-1 to sub channel- 2.



Continuity: $\frac{dm_i}{dx} = -w$ Axial momentum: $\frac{d}{dx}(m_1u_1) + w[H(w)u_1 + h(-w)u_2] =$

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$-F_1 - A_1 \frac{dp_1}{dx}$

Energy : $\frac{d}{dx}(m_1 h_1) = q_1 - w[H(w)h_1 + H(-w)h_2]$

Analogous equations for sub channel 2 can be obtained from these by substituting w for w and by interchanging subscripts 1 and 2. In this equation set, H is the Heavy side unit step function. F represents pressure loss per unit length due to friction, A is the cross-sectional area, q represents the heat energy added per unit length, and the variables, u, p and h stand for particle velocity, pressure and enthalpy respectively. In analogy with the pressure drop due to friction in a long pipe, a lateral momentum balance may be taken as $p1 - p2 = Cw[W]$, where C is a cross- flow friction factor. To simplify the above equation, the following assumptions are made. Cross- sectional area is constant; the coolant is incompressible; there is no enthalpy change; and the frictional pressure loss function is of the form $1 = m_1 u_1 F$, where F is a constant. With these assumptions, the equations may be combined and written in the following form:

$$\frac{dm_1}{dx} = -w$$

$$\frac{d}{dx}(w|w|) = \epsilon^{-1}\{\frac{1-2m_1}{2} + 2w[1 - H(w)m_1 + H(-w)(m_1 - 1)]\}$$

To make the above system into the symmetric form, take

$$x = m_1 - \frac{1}{2}, y = \frac{w}{2}, t = x$$

Hence we get

$$\frac{dx}{dt} = -2y$$

Replacing x by x_1 and y by x_2 , we have

$$\dot{x}_1 = -2x_2$$

$$\frac{d}{dt}(x_2|x_2|) = (4\epsilon)^{-1}[x_1 + 2(x_2 - 2x_1|x_2|)]$$

An analysis is carried out in four different ways depending upon the values of x_2 and ϵ as given below :

I. $x_2 > 0 \; and \; \epsilon \neq 0$

II. $x_2 < 0 \; and \; \epsilon \neq 0$

III. $x_2 > 0 \; and \; \epsilon = 0$

(i) $x_2 < 0 \; and \; \epsilon = 0$

In the first two cases the parameter has been varied from $10^0, 10^1, 10^2, \ldots 10^7$ and in the last two cases, has been set to zero.

Case (i)

When $x_2 > 0 \; and \; \epsilon \neq 0$ In this case equation becomes

$$\dot{x}_1 = -2x_2$$

$$8\epsilon x_2 \dot{x}_2 = -x_1 + 2x_2 - 4x_1 x_2$$

The above two equations can be considered as a system of equations of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 8\epsilon x_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4x_1 x_2 \end{bmatrix}$$

This is of the form (Evans et al. [48])

$$K(x(t))\dot{x}(t) = Ax(t) + f(x(t))$$

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The above first order non-linear system, representing the highly simplified two channel model of a nuclear reactor core from uid dynamics, when $x_2 > 0$ *and* $\epsilon \neq 0$, can be converted into a second order equation in order to reduce the number of equations, as well as the number of unknowns, and is given as

$$\ddot{x}_1 = \frac{1}{2\epsilon}[\frac{-x_1}{\dot{x}_1} - 1 + 2x_1]$$

Where

$$x_2 = \frac{-\dot{x}_1}{2}$$

Hence the above equation is of the form

$$\ddot{x}_1 = \emptyset(\epsilon) f(t, x_1, \dot{x}_1)$$

Where

$$\emptyset(\epsilon) = \frac{1}{2\epsilon}$$

Case (ii)

When $x_2 < 0$ *and* $\epsilon \neq 0$

In this case equation becomes

$$\dot{x}_1 = 2x_2$$
$$8\epsilon x_2 \dot{x}_2 = -x_1 + 2x_2 - 4x_1 x_2$$

The above two equations can be considered as a system of equations of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 8\epsilon x_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4x_1 x_2 \end{bmatrix}$$

This is of the form (Evans et al. [48])

$$K(x(t))\dot{x}(t) = Ax(t) + f(x(t))$$

The above first order non-linear system, representing the highly simplified two channel model of a nuclear reactor core from uid dynamics, when $x_2 < 0$ *and* $\epsilon \neq 0$ can be converted into a second order equation in order to reduce the number of equations, as well as the number of unknowns, and is given as

$$\ddot{x}_1 = \frac{1}{2\epsilon}[\frac{x_1}{\dot{x}_1} + 1 + 2x_1]$$

Where

$$x_2 = \frac{\dot{x}_1}{2}$$

Hence the above equation is of the form

$$\ddot{x}_1 = \emptyset(\epsilon) f(t, x_1, \dot{x}_1)$$

Where

$$\emptyset(\epsilon) = \frac{1}{2\epsilon}$$

Case (iii)

When $x_2 > 0$ *and* $\epsilon = 0$ In this case equation becomes

$$\dot{x}_1 = -2x_2$$
$$0 = -x_1 + 2x_2 - 4x_1 x_2$$

The above two equations can be considered as a system of equations of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4x_1 x_2 \end{bmatrix}$$

The above system is a singular non-linear system and it is of the form

$$K(x(t))\dot{x}(t) = Ax(t) + f(x(t))$$

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This system can be written as

$$\ddot{x}_1 = \frac{x_1}{2x_1 - 1}$$

Where

$$x_2 = \frac{-\dot{x}_1}{2}$$

The above equations has been converted into a second order equations as

$$\ddot{x}_1 = \frac{-\dot{x}_1}{(2x_1 - 1)^2}$$

Where

$$x_2 = \frac{-\dot{x}_1}{2}$$

Hence the above equation is of the form

$$\ddot{x}_1 = f(t, x_1, \dot{x}_1)$$

Case (IV)

When $x_2 < 0 \; and \; \epsilon = 0$

In this case equation becomes

$$\dot{x}_1 = 2x_2$$
$$0 = -x_1 - 2x_2 - 4x_1x_2$$

The above two equations can be considered as a system of equations of the form

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -4x_1x_2 \end{bmatrix}$$

The above system is a singular non-linear system and it is of the form

$$K(x(t))\dot{x}(t) = Ax(t) + f(x(t))$$

This system can be written as

$$\ddot{x}_1 = \frac{x_1}{2x_1 + 1}$$

Where

$$x_2 = \frac{\dot{x}_1}{2}$$

The above equations has been converted into a second order equations as

$$\ddot{x}_1 = \frac{-\dot{x}_1}{(2x_1 + 1)^2}$$

Where

$$x_2 = \frac{-\dot{x}_1}{2}$$

Hence the above equation is of the form

$$\ddot{x}_1 = f(t, x_1, \dot{x}_1)$$

LEAPFROG METHOD FOR NON-LINEAR SINGULAR SYSTEM FROM FLUID DYNAMICS

In this section we modified the method to solve the non-Linear singular system from fluid dynamics, as follows; Euler's Method approximates the derivative in the form of $y' = f(t, y), y(t_0) = y_0, y \in R^d$ by a finite difference quotient$y' \approx \frac{y(t+h)-y(t)}{h}$.. We shall usually discretize the independent variable in equal increments:

$$t_{n+1} = t_n + h, n = 0,1,2, \dots t_0$$

Henceforth we focus on the scalar case, $N = 1$. Rearranging the difference quotient gives us the corresponding approximate values of the dependent variable:

$$y_{n+1} = y_n + hf(t_n, y_n), n = 0,1,2, \dots t_0$$

To obtain the leapfrog method, we discrete t_n as in $t_{n+1} = t_n + h, n = 0,1,2, \dots t_0$ but we double the time interval, h, and write the midpoint approximation

$y(t+h) - y(t) \approx hy'\left(t + \frac{h}{2}\right)$

in the form

$$y'^{(t+h)} \approx \frac{y(t+2h) - y(t)}{h}$$

and then discrete it as follows:

$$y_{n+1} = y_{n-1} + 2hf(t_n, y_n), n = 0,1,2, \dots t_0$$

The leapfrog method is a linear m = 2-step method, with $a_0 = 0$; $a_1 = 1$; $b_{\ 1} = 1$; $b_0 = 2$ and $b_1 = 0$. It uses slopes evaluated at odd values of n to advance the values at points at even values of n, and vice versa, reminiscent of the childrens game of the same name. For the same reason, there are multiple solutions of the leapfrog method with the same initial value $y = y_0$. This situation suggests a potential instability present in multistep methods, which must be addressed when we analyze them two values, y_0 and y_1, are required to initialize solutions of $y_{n+1} = y_n + hf(t_n, y_n),\ n = 0,1,2, \dots t_0$ uniquely, but the analytical problem $y' = f(t, y), y(t_0) = y_0, y \in R^d$ only provides one. Also for this reason, one-step methods are used to initialize multistep methods. In order to illustrate the possible practical use of this method we apply the above technique to the following examples of non-Linear singular system from fluid dynamics.

DISCRETE SOLUTIONS FOR NON-LINEAR SINGULAR SYSTEM FROM FLUID DYNAMICS

It is very difficult to obtain the exact solution of this non-linear equation. Hence it has been analyzed by the following numerical methods by the way of determining the discrete solutions at different time intervals:

I. Single-term Haar wavelet series.
II. Leapfrog method.

The above method has been applied to determine the approximate solutions for all four cases of the nuclear reactor core problem discussed in section The discrete solutions of a two channel model of a nuclear reactor core problem for the cases (i) when $x_2 > 0$ and 6= 0 (ii) when $x_2 < 0$ and 6= 0 [i.e., Eq.(8.6) and Eq. (8.8)] have been determined using the single-term Haar wavelet series method by varying the parameter from 10^0 to 10^7 with $x_1(0) = 1$; $x_1(0) = 1$ and the results are given in the Tables 1 - 4 and the discrete solution for the cases (iii) when $x_2 > 0$ and = 0 (iv) when $x_2 < 0$ and = 0 [i.e., singular systems] have been determined using single-term Haar wavelet series method with $x_1(0) = 1$; $x_1(0) = 1$ and the results are given in the Tables 1 – 2.

CONCLUSIONS

The nuclear reactor core problem has been studied under four different cases (specified section Tables) by way of determining the discrete solutions for different time t using the single-term Haar wavelet series method and Leapfrog method. In for the same problem, the approximate solution was determined using Leapfrog method and it was mentioned that the single-term Haar wavelet series method failed to obtained approximate solutions when the parameter 10^3. But in this chapter 6, it has been established that the single-term Haar wavelet series method are adequate enough to determine approximate solutions for all values of (i.e., $\in = 0, 10^0, 10^1; \dots, 10^7$).

In cases (iii) and (iv), when = 0, the system reduces to a singular system for both $x_2 > 0$ and $x_2 < 0$. It is observed that, for a singular system, the discrete solutions obtained by the single-term Haar wavelet series method and Leapfrog method are found to be similar (refer Tables 1 - 12). However, for the cases (i) when $x_2 > 0$ and 6= 0 (ii) when $x_2 < 0$ and 6= 0, it has been noted that the discrete solutions, obtained by employing the discussed the single-term Haar wavelet series method and Leapfrog method, coincide with each other (refer Tables 1 -12). When 10^6, the discrete solution obtained for the nuclear reactor core problem converges and remains stable.

Hence, by comparing the results obtained for the nuclear reactor core problem discussed under four cases; the Leapfrog method is more suitable for studying the nuclear reactor core problem.

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Table 1: Solutions of equation Case (i) and STHWS method for x_1

t	ε=10^0	ε=10^1	ε=10^2	ε=10^3
0	1.0000	1.0000	1.0000	1.0000
0.5	0.5000	1.5011	1.5001	1.5000
1	2.1035	2.0085	2.0008	2.0001
1.5	2.9072	2.5293	2.5028	2.5003
2	4.1356	3.0711	3.0067	3.0007
2.5	6.1192	3.6427	3.5131	3.5013
3	9.3819	4.2544	4.0228	4.0023
3.5	14.7670	4.9183	4.5363	4.5363
4	23.6559	5.6486	5.0545	5.0545
4.5	38.3217	6.4619	5.5779	5.5779
5	62.5103	7.3777	6.1074	6.1074
t	ε=10^4	ε=10^5	ε=10^6	ε=10^7
0	1	1	1	1
0.5	1.5	1.5	1.5	1.5
1	2.00001	2	2	2
1.5	2.50003	2.5	2.5	2.5
2	3.00007	3	3	3
2.5	3.50013	3.50001	3.5	3.5
3	4.00022	4.00002	4	4
3.5	4.50036	4.50003	4.50001	4.50001
4	5.00053	5.00004	5.00002	5.00002
4.5	5.50076	5.50006	5.50003	5.50003
5	6.00104	6.0001	6.00004	6.00004

t	= 10^0	= 10^1	= 10^2	= 10^3
0	1	1	1	1
0.5	0.5	1.50105	1.5001	1.5
1	2.103502	2.00852	2.00084	2.00008
1.5	2.907169	2.5293	2.50282	2.50028
2	4.13519	3.07107	3.00671	3.00067
2.5	6.119246	3.64267	3.51314	3.5013
3	9.381907	4.25438	4.02279	4.00225
3.5	14.767	4.91829	4.53632	4.53632
4	23.65592	5.6486	5.05446	5.05446
4.5	38.32171	6.46194	5.57791	5.57791
5	62.5103	7.37769	6.10744	6.10744

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	1.5	1.5	1.5	1.5
1	2.00001	2	2	2
1.5	2.50003	2.5	2.5	2.5
2	3.00007	3	3	3
2.5	3.50013	3.50001	3.5	3.5
3	4.00022	4.00002	4	4
3.5	4.50036	4.50003	4.50001	4.50001
4	5.00053	5.00004	5.00002	5.00002
4.5	5.50076	5.50006	5.50003	5.50003

5	6.00104	6.0001	6.00004	6.00004
0	1	1	1	1

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	1.5	1.5	1.5	1.5
1	2.00001	2	2	2
1.5	2.50003	2.5	2.5	2.5
2	3.00007	3	3	3
2.5	3.50013	3.50001	3.5	3.5
3	4.00022	4.00002	4	4
3.5	4.50036	4.50003	4.50001	4.50001
4	5.00053	5.00004	5.00002	5.00002
4.5	5.50076	5.50006	5.50003	5.50003
5	6.00104	6.0001	6.00004	6.00004

Table 2: Solutions of equation Case (i) and STHWS method for x_2

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	-0.5354	-0.5032	-0.5003	-0.5
1	-0.6702	-0.5129	-0.5013	-0.5001
1.5	-0.9721	-0.5299	-0.5028	-0.5003
2	-1.5381	-0.5551	-0.5051	-0.5005
2.5	-2.515	-0.5898	-0.5079	-0.5008
3	-4.1475	-0.6356	-0.5115	-0.5011
3.5	-6.8473	-0.6946	-0.5157	-0.5015
4	-11.3	-0.7688	-0.5207	-0.502
4.5	-18.64	-0.8611	-0.5264	-0.5025
5	-30.741	-0.9742	-0.5328	-0.5031

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	-0.5	-0.5	-0.5	-0.5
1	-0.5	-0.5	-0.5	-0.5
1.5	-0.5	-0.5	-0.5	-0.5
2	-0.5001	-0.5001	-0.5	-0.5
2.5	-0.5001	-0.5	-0.5	-0.5
3	-0.5001	-0.5	-0.5	-0.5
3.5	-0.5002	-0.5	-0.5	-0.5
4	-0.5002	-0.5	-0.5	-0.5
4.5	-0.5003	-0.5	-0.5	-0.5
5	-0.5003	-0.5	-0.5	-0.5

Table 3: Solutions of equation Case (ii) and STHWS method for x_1

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	1.77025	1.52794	1.50281	1.50028
1	3.22725	2.12374	2.01248	2.00125
1.5	5.74391	2.80613	2.53089	2.50309
2	9.96764	3.59529	3.05989	3.0056
2.5	16.9811	4.51351	3.60137	3.51015
3	28.578	5.58583	4.15721	4.01575
3.5	47.7207	6.84079	4.7293	4.52296
4	79.297	8.3112	5.31956	5.03199
4.5	131.368	10.0351	5.92991	5.54302
5	217.224	12.0566	6.56227	6.05624

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Table 4: Solutions of equation Case (ii) and STHWS method for x_2

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	1.50003	1.5	1.5	1.5
1	2.00013	2.00001	2	2
1.5	2.50031	2.50003	2.5	2.5
2	3.0006	3.00006	3	3
2.5	3.50102	3.5001	3.5	3.5
3	4.00158	4.00016	4	4
3.5	4.5023	4.50023	4.50001	4.50001
4	5.0032	5.00032	5.00002	5.00002
4.5	5.5043	5.50004	5.50003	5.50003
5	6.00563	6.00056	6.00004	6.00004

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	1.06923	0.55884	0.50593	0.50059
1	1.90565	0.633359	0.513728	0.501375
1.5	3.23294	0.73226	0.52339	0.50234
2	5.39053	0.84977	0.53493	0.5035
2.5	8.92805	0.99083	0.54834	0.50484
3	14.7473	1.15855	0.56365	0.50637
3.5	24.3327	1.35681	0.58086	0.50809
4	40.1302	1.59033	0.59998	0.51
4.5	66.1718	1.86475	0.62104	0.51209
5	109.104	2.18678	0.64407	0.51437

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	0.50006	0.50001	0.5	0.5
1	0.50014	0.50001	0.5	0.5
1.5	0.50023	0.50002	0.5	0.5
2	0.50035	0.50004	0.5	0.5
2.5	0.50048	0.50005	0.5	0.5
3	0.50064	0.50006	0.5	0.5
3.5	0.50081	0.50008	0.50001	0.5
4	0.501	0.5001	0.50001	0.5
4.5	0.50121	0.50012	0.50001	0.5
5	0.50144	0.50014	0.50001	0.5

Table 8.5: Solutions of equation Case (iii) and STHWS method for x_1 and x_2

t	= 10^4	= 10^5
0	1	1
0.5	1.42821	-0.3847
1	1.791537	-0.34678
1.5	2.127465	-0.32681
2	2.447541	-0.31418
2.5	2.757087	-0.30538
3	3.059052	-0.29885
3.5	3.355264	-0.29378
4	3.646944	-0.2872
4.5	3.934948	-0.28639
5	4.21991	-0.2836

Table 8.6: Solutions of equation Case (iv) and STHWS method for x_1 and x_2

t	X_1	X_2
0	1	1

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0.5	1.488729	0.479521
1	1.961758	0.467443
1.5	2.424938	0.459403
2	2.881311	0.453635
2.5	3.332675	0.449281
3	3.780187	0.449281
3.5	4.224638	0.443124
4	4.666598	0.44086
4.5	5.106484	0.438962
5	5.544618	0.437346

Table 8.7: Solutions of equation Case (i) and Leapfrog method for x_1

t	$= 10^0$	$= 10^1$	$= 10^2$	$= 10^3$
0	1	1	1	1
0.5	0.500001	1.50105	1.50019	1.50009
1	2.103502	2.00852	2.00084	2.00008
1.5	2.907169	2.52939	2.50282	2.50028
2	4.135199	3.07107	3.00671	3.00067
2.5	6.119246	3.64267	3.51314	3.5013
3	9.381907	4.25438	4.02279	4.00225
3.5	14.76799	4.91829	4.53632	4.53632
4	23.65592	5.64869	5.05446	5.05446
4.5	38.32171	6.46194	5.57791	5.57791
5	62.51039	7.37769	6.10744	6.10744

t	$= 10^4$	$= 10^5$	$= 10^6$	$= 10^7$
0	1	1	1	1
0.5	1.50009	1.5	1.5	1.5
1	2.00001	2	2	2
1.5	2.50003	2.5	2.5	2.5
2	3.00007	3	3	3
2.5	3.50013	3.50001	3.5	3.5
3	4.00022	4.00002	4	4
3.5	4.50036	4.50003	4.50001	4.50001
4	5.00053	5.00053	5.00053	5.00053
4.5	5.50076	5.50006	5.50003	5.50003
5	6.00104	6.00019	6.00004	6.00004

Table 8.8: Solutions of equation Case (i) and Leapfrog method for x_2

t	$= 10^0$	$= 10^1$	$= 10^2$	$= 10^3$
0	1	1	1	1
0.5	-0.535368	-0.50316	-0.50031	-0.500003
1	-0.670201	-0.51292	-0.50125	-0.500125
1.5	-0.972082	-0.52991	-0.50283	-0.500281
2	-1.538084	-0.55519	-0.50505	-0.500501
2.5	-2.515008	-0.58989	-0.50793	-0.500782
3	-4.147465	-0.63563	-0.51148	-0.501127
3.5	-6.847252	-0.69456	-0.51572	-0.501535

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4	-11.30009	-0.76884	-0.52067	-0.502007
4.5	-18.64044	-0.86107	-0.52636	-0.502542
5	-30.74066	-0.97417	-0.53282	-0.503141

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	-0.50003	-0.5	-0.5	-0.5
1	-0.50001	-0.5	-0.5	-0.5
1.5	-0.50003	-0.5	-0.5	-0.5
2	-0.50005	-0.50005	-0.5	-0.5
2.5	-0.50008	-0.50001	-0.5	-0.5
3	-0.50011	-0.50001	-0.5	-0.5
3.5	-0.50015	-0.50001	-0.5	-0.5
4	-0.50029	-0.50002	-0.5	-0.5
4.5	-0.50025	-0.50003	-0.5	-0.5
5	-0.50031	-0.50003	-0.5	-0.5

Table 8.9: Solutions of equation Case (ii) and Leapfrog method for x_1

T	= 10^0	= 10^1	= 10^2	= 10^3
0	1	1	1	1
0.5	1.7702536	1.5279384	1.50281	1.500281
1	3.2272538	2.1237375	2.012484	2.001255
1.5	5.7439054	2.8061257	2.530888	2.503093
2	9.9676389	3.5952898	3.059891	3.005599
2.5	16.981142	4.5135187	3.601371	3.510154
3	28.578028	5.5858293	4.15721	4.015746
3.5	47.720726	6.8407866	4.729303	4.522964
4	79.296951	8.3112018	5.319559	5.031993
4.5	131.36757	10.035072	5.929907	5.543022
5	217.22435	12.056558	6.562268	6.056237

t	= 10^4	= 10^5	= 10^6	= 10^7
0	1	1	1	1
0.5	1.500028	1.5000035	1.5	1.5
1	2.000125	2.0000122	2	2
1.5	2.500319	2.5000317	2.5	2.5
2	3.000699	3.0000649	3	3
2.5	3.501016	3.5001018	3.5	3.5
3	4.001575	4.0001572	4	4
3.5	4.502297	4.5002298	4.50001	4.50001
4	5.003299	5.0003189	5.00002	5.00002
4.5	5.504303	5.5000427	5.50003	5.50003
5	6.005626	6.0005599	6.00004	6.00004

Table 8.10: Solutions of equation Case (ii) and Leapfrog method for x_2

t	= 10^0	= 10^1	= 10^2	= 10^3
0	1	1	1	1
0.5	1.0692324	0.5588483	0.505932	0.5005946
1	1.9056584	0.6333587	0.513728	0.5013750

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1.5	3.2329445	0.7322594	0.523392	0.5023438
2	5.3905289	0.8497732	0.534929	0.5034992
2.5	8.9280521	0.9908322	0.548344	0.5048437
3	14.747292	1.1585519	0.563649	0.5063731
3.5	24.332659	1.3568148	0.580855	0.5080929
4	40.130196	1.5903316	0.599979	0.5099982
4.5	66.171783	1.8647593	0.621044	0.5120917
5	109.10431	2.1867828	0.644074	0.5143724
t	$= 10^4$	$= 10^5$	$= 10^6$	$= 10^7$
0	1	1	1	1
0.5	0.500059	0.5000066	0.5	0.5
1	0.500138	0.5000141	0.5	0.5
1.5	0.500234	0.5000234	0.5	0.5
2	0.500356	0.5000355	0.5	0.5
2.5	0.500484	0.5000491	0.5	0.5

Table 8.11: Solutions of equation Case (iii) and Leapfrog method for x_1 and x_2

t	X_1	X_2
0	1	1
0.5	1.428211	-0.384
1	1.791537	-0.346
1.5	2.127465	-0.3268
2	2.447541	-0.3053
2.5	2.757087	-0.3053
3	3.059052	-0.2863
3.5	3.355264	-0.2937
4	3.646944	-0.2863
4.5	3.934948	-0.2863
5	4.219906	-0.2831

Table 8.12: Solutions of equation Case (iv) and Leapfrog method for x_1 and x_2

t	X_1	X_2
0	1	1
0.5	1.42729	0.479521
1	1.96175	0.467443
1.5	2.42493	0.459403
2	2.88131	0.453635
2.5	3.33267	0.449281
3	3.78018	0.445871
3.5	4.22463	0.445871
4	4.66659	0.440865
4.5	5.10648	0.438962
5	5.54461	0.437346

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